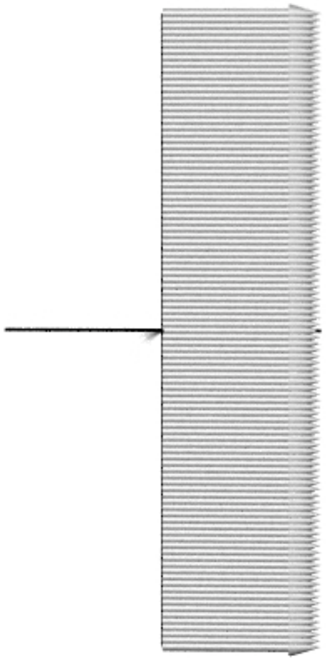
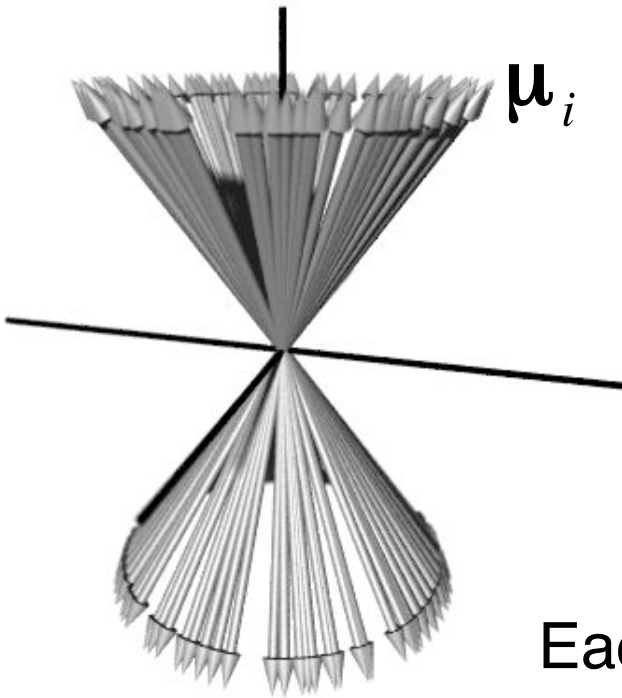
Spin gymnastics

- Spin packets
- RF pulses
- The rotating frame
- Signal detection
- Magnetic field gradients
- Spin echoes



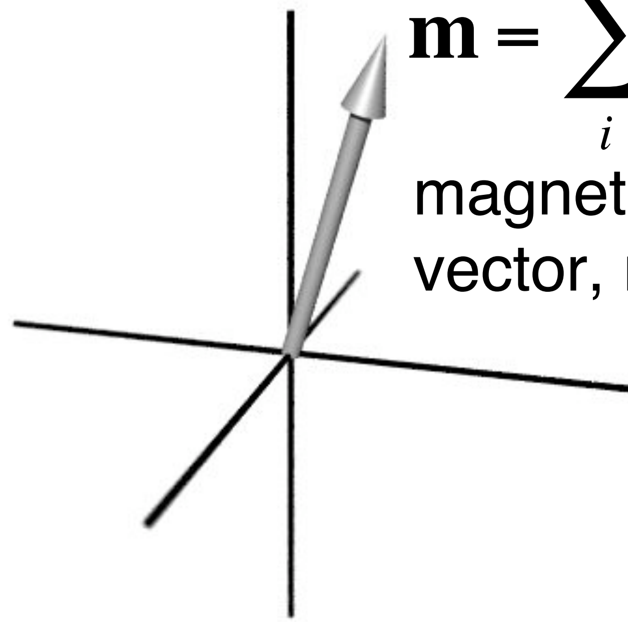
Spin packet - isochromat

Ensemble of spins experiencing the same magnetic field



$\boldsymbol{\mu}_i$

$$\mathbf{m} = \sum_i \boldsymbol{\mu}_i$$



magnetization vector, **m**

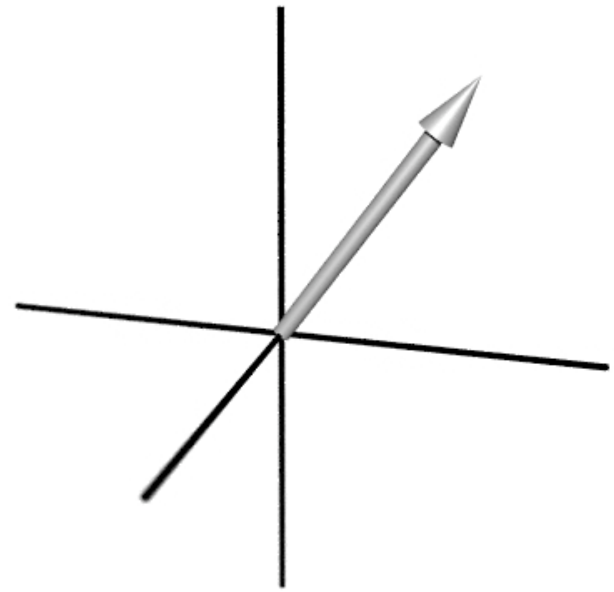
Each $\boldsymbol{\mu}_i$ obeys quantum laws, but **m** behaves classically!

Free precession

cf. spinning top

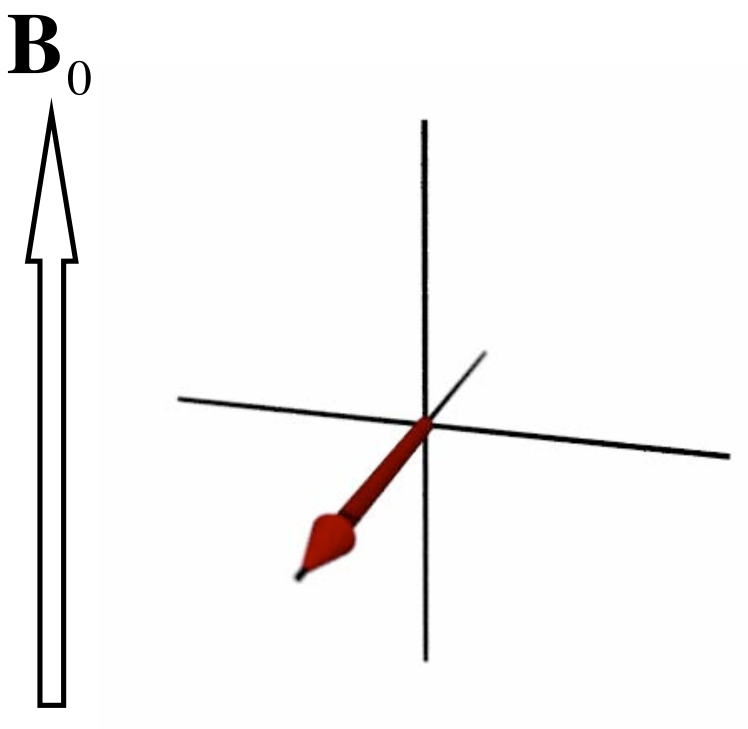
$$\frac{\mathrm{d}\mathbf{m}(t)}{\mathrm{d}t} = -\gamma \mathbf{B}_0 \times \mathbf{m}(t)$$

$$\omega_0 = -\gamma B_0$$



Radiofrequency (RF) field, $\mathbf{B}_1$

- Magnetic field $\mathbf{B}_1$ ($B_1 \ll B_0$) rotating in *xy*-plane with frequency ω_{RF}
- Produced by the RF coil



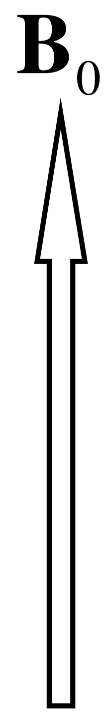
Resonance

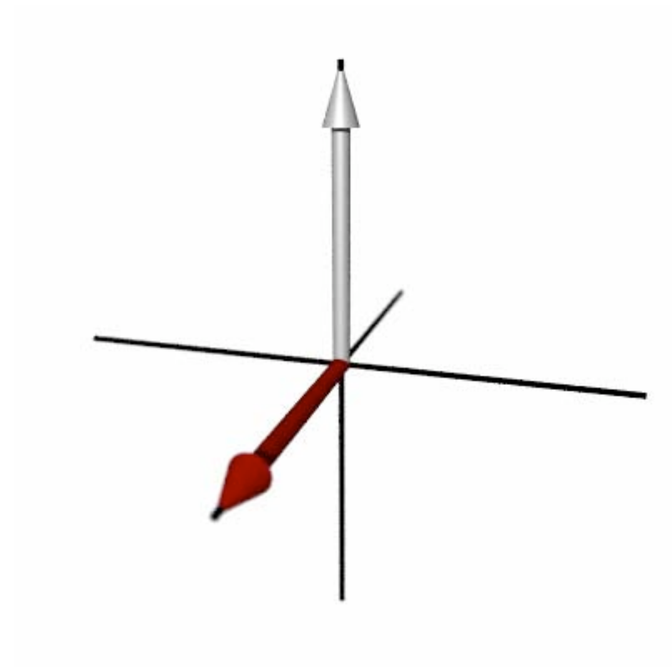
- **m** tilted from z-axis
  if $\omega_{\mathrm{RF}} \approx \omega_0$
- Resonance!

  freq. of perturbation =
  some natural freq.
  of the system

$$\frac{\mathrm{d}\mathbf{m}(t)}{\mathrm{d}t} = -\gamma \mathbf{B}(t) \times \mathbf{m}(t)$$

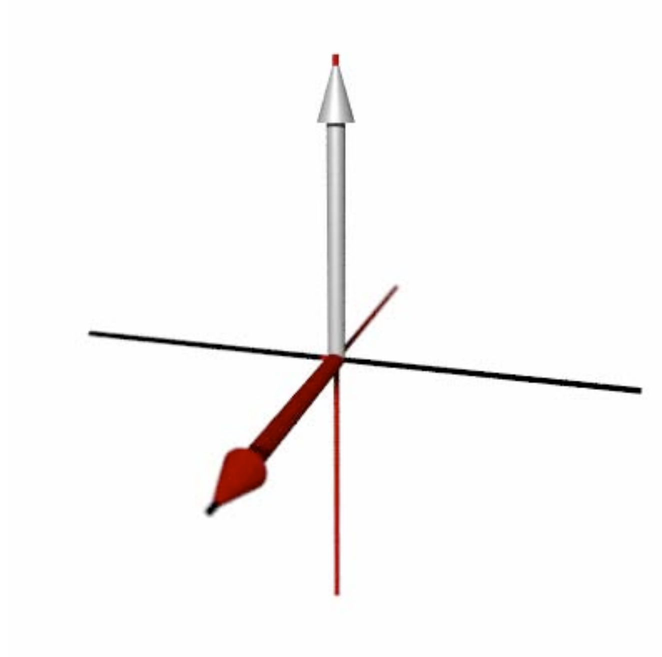
$$\mathbf{B}(t) = \mathbf{B}_0 + \mathbf{B}_1(t)$$





Rotating frame - lab view

- Reference frame rotating in *xy*-plane with frequency ω_{RF}



seen from the lab

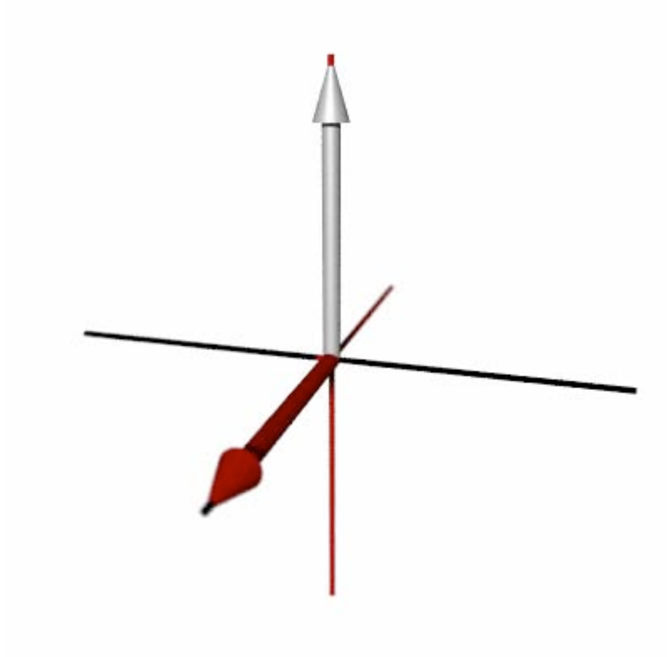
Step into the rotating frame

- Motion of **m** appears simpler: rotation of **m** around $\mathbf{B}_1$ with freq. ω_1

$$\omega_1 = -\gamma B_1$$

nutation frequency, ω_1

rotating frame often used implicitly



seen from the rotating frame

RF pulse

- Short burst of RF radiation (a few μs)

$$\alpha = \omega_1 t_{\mathrm{RF}}$$

nutation angle, α
pulse length, t_{RF}

flip angle ($-\alpha$)

$$90°_x$$

RF phase
(i.e. axis of rotation in the rotating frame)

Calculate the Cartesian components of the magnetization after the following RF pulses applied to thermal equilibrium magnetization:

a) $90°_x$

b) $90°_y$,

c) $180°_x$

d) $180°_y$

What RF pulse would give the following rotation:

a) (1, 0, 0) → (-1, 0, 0)

b) (1, 0, 0) → (0, 0, 1)

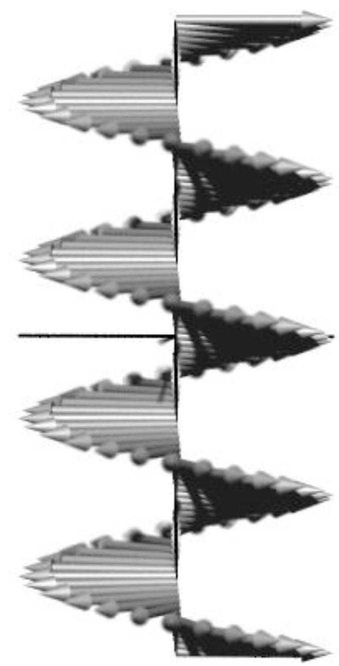
c) (1, 0, 0) → (0, 1, 0)

Macroscopic magnetization, **M**

$$\mathbf{M}(t) = \int \rho(\mathbf{r})\mathbf{m}(\mathbf{r},t)d\mathbf{r}$$

time, t
position, **r**
spin density, ρ

integral over entire sample



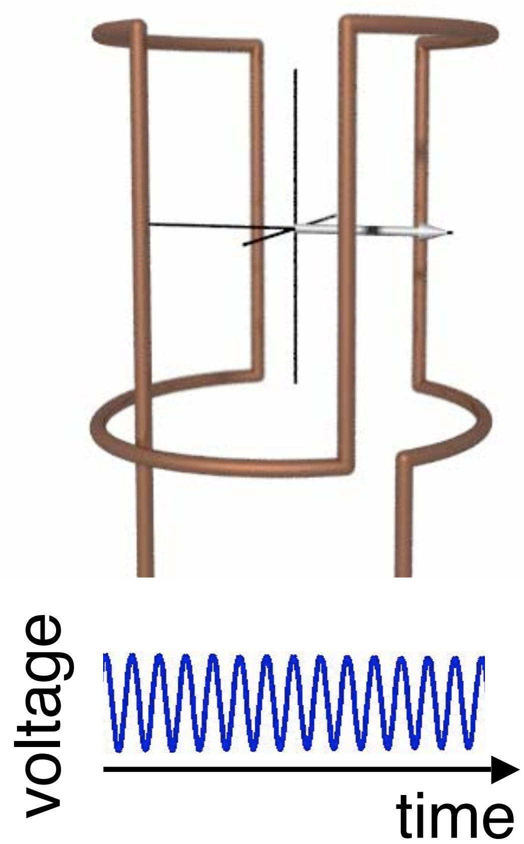
$\mathbf{m}(\mathbf{r},t)$

Signal detection

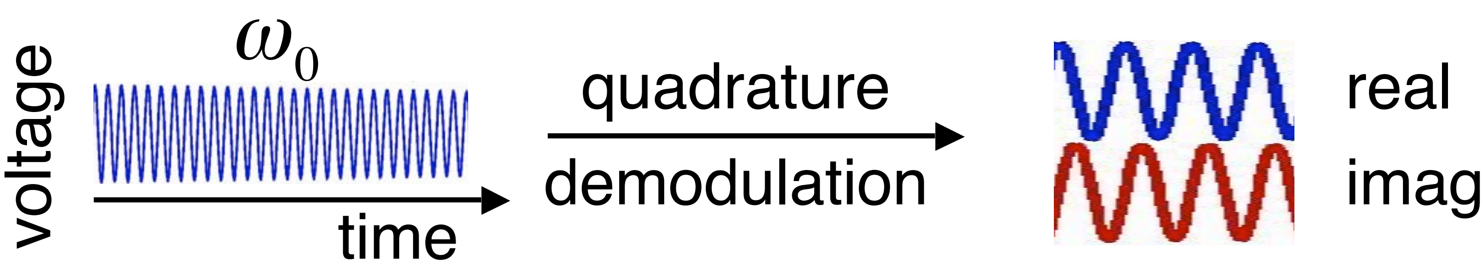
- Rotating magnetization => alternating voltage in the coil

$$S \propto M_{xy}$$

signal, *S*
transverse magnetization, M_{xy}



Detection in the rotating frame



$$\Delta\omega_0 = \omega_0 - \omega_{\mathrm{RF}}$$

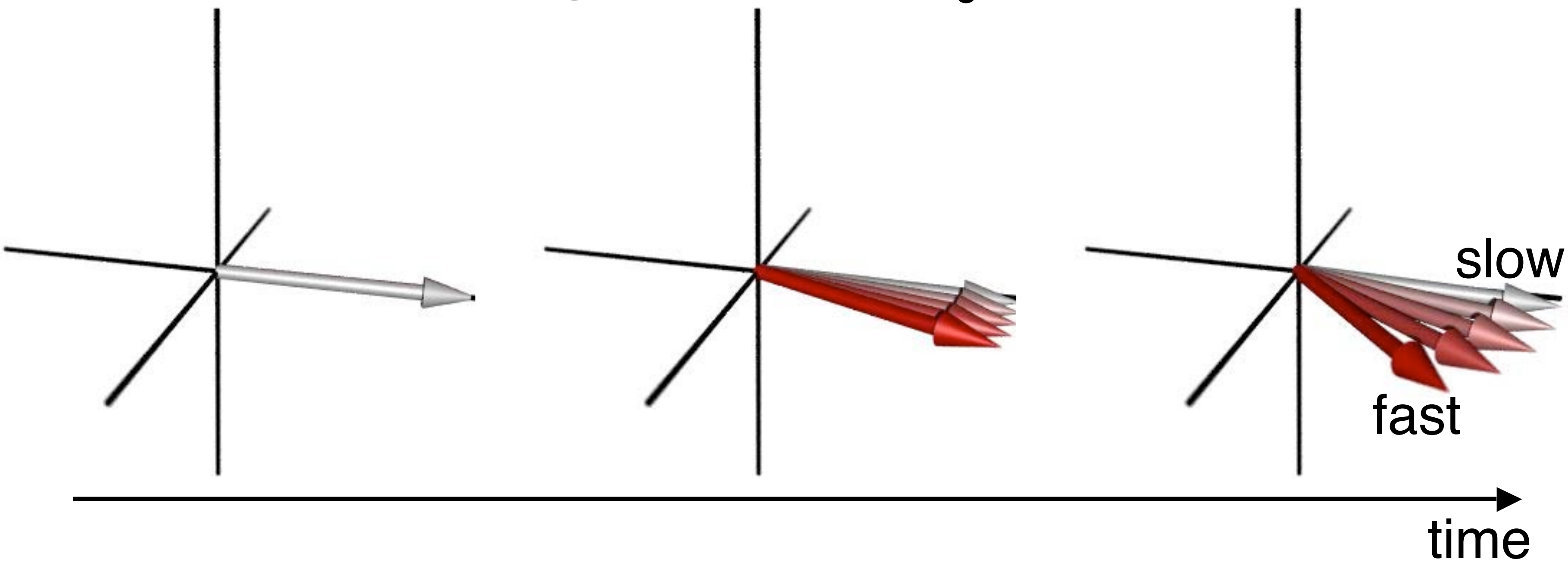
offset frequency, $\Delta\omega_0$

- Real and imaginary parts of the signal *S* correspond to M_x and M_y in the rotating frame

$$S(t) \propto M_x(t) + iM_y(t)$$

Loss of coherence

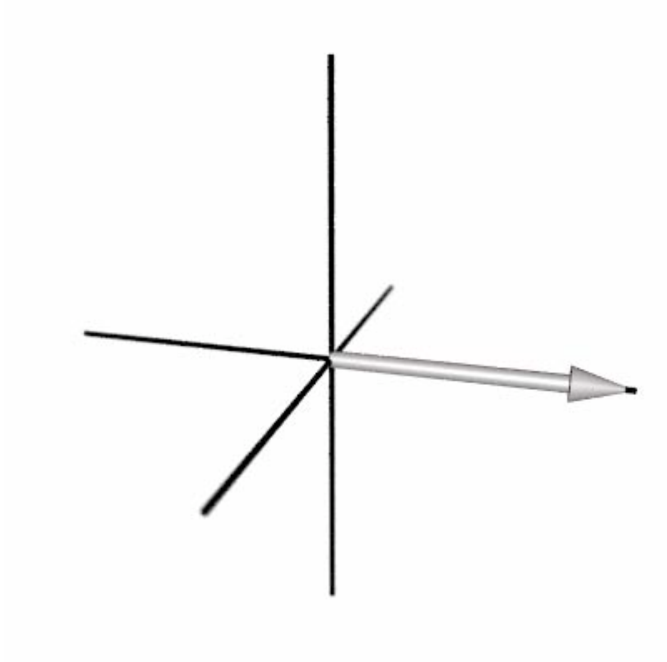
- Different $\Delta\omega_0$ for spin packets experiencing different B_0 or σ



Refocusing by 180° pulses

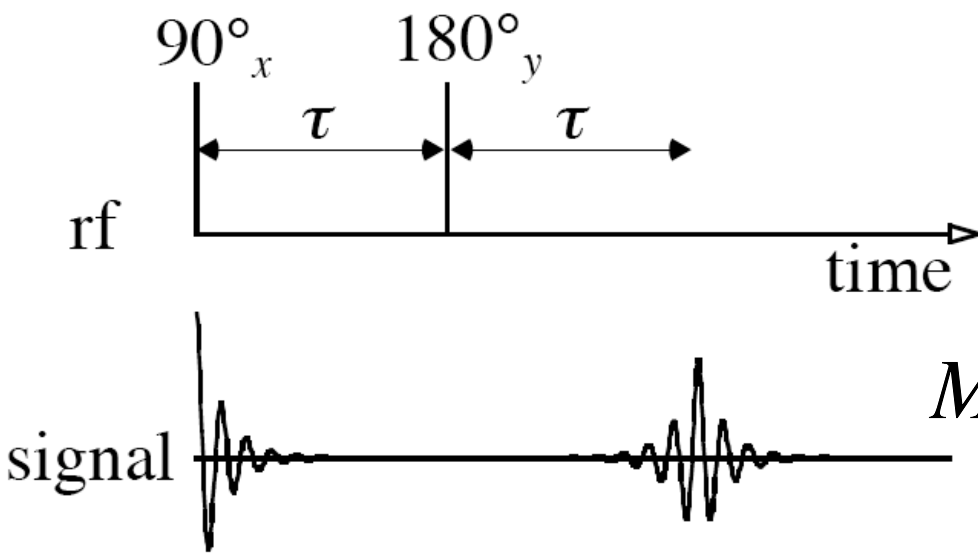
Spin echo!

Sketch $S(t)$



Spin echo pulse sequence

- a.k.a. Hahn echo
- T_2 measurement
- 90° - τ - 180° - τ - acq



$$M_{xy}(2\tau) = M_0 \exp\left(-\frac{2\tau}{T_2}\right)$$

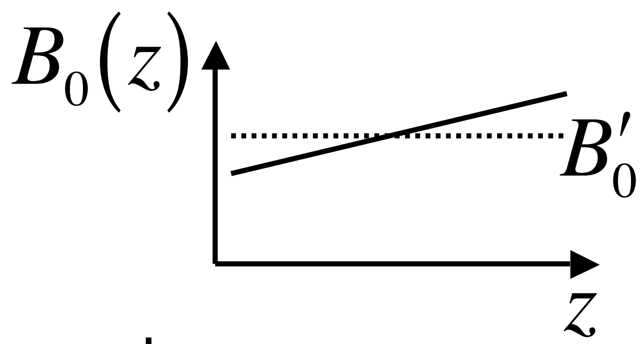
Magnetic field gradients, **G**

- Inhomogeneous magnetic fields

gradient vector

$$B_0(\mathbf{r}) = B_0' + \mathbf{G} \cdot \mathbf{r}$$

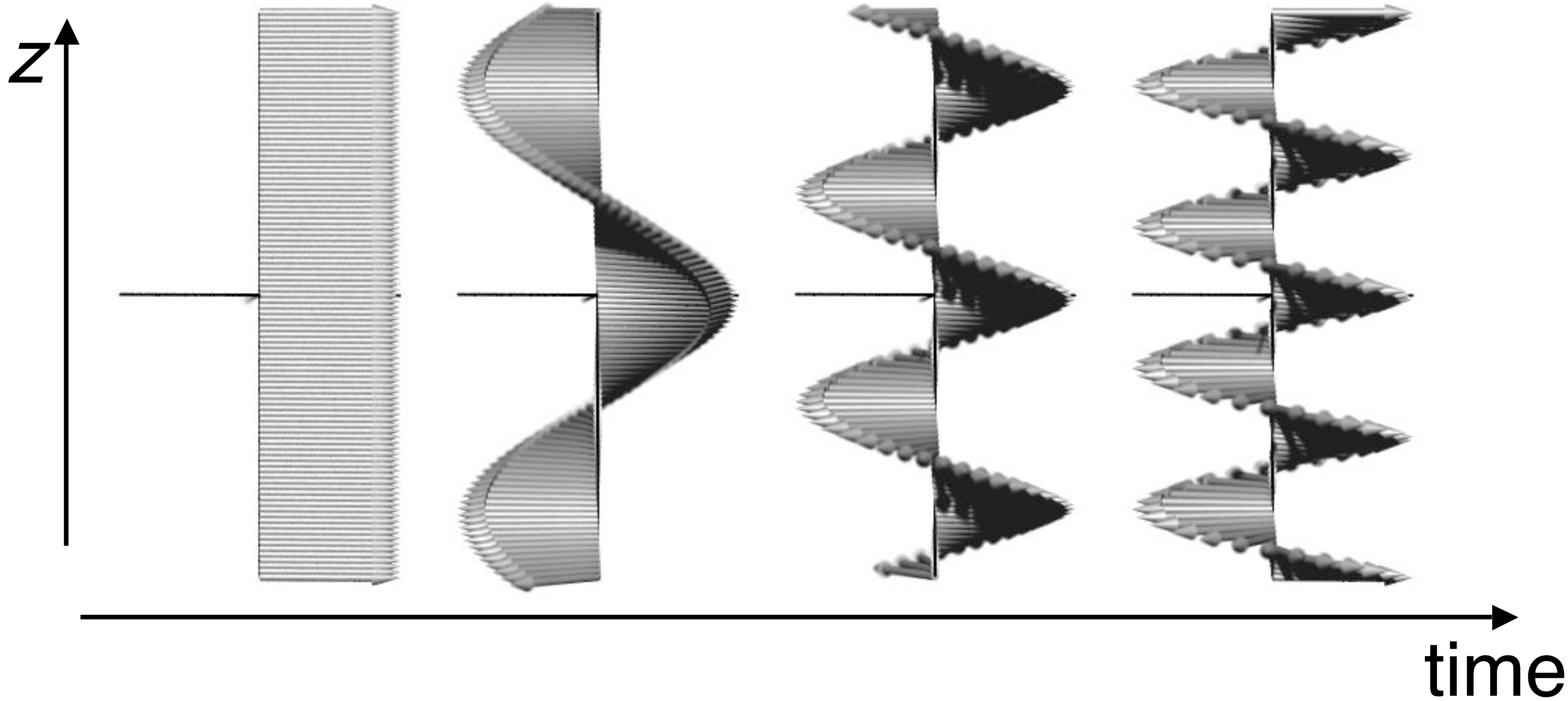
homogeneous component



$$\left.\begin{aligned} B_0(z) &= B_0' + Gz \\ \omega_{\mathrm{RF}} &= -\gamma B_0' \end{aligned}\right\} \quad \Delta\omega_0(z) = -\gamma G z$$

Spin evolution in a gradient

$$\Delta\omega_0(z) = -\gamma G z$$



Under what conditions would the spins behave in the following way?